**Multi-sensor measurement of the Earth's magnetic field**

The Earth's magnetic field is not constant and can be influenced by various factors. Solar activities such as coronal mass ejections, solar flares, and other phenomena can cause interactions between the solar wind and the Earth's magnetic field, leading to geomagnetic storms and other effects. These events can interfere with power grids, communication systems, navigation, and other technologies to varying degrees. At the same time, the movement of the liquid inner core, as well as the interactions between the outer core and the mantle, can also lead to changes in the magnetic field. To better understand these variations and their potential impacts, it is necessary to establish geomagnetic observatories to continuously monitor the changes in the magnetic field, providing strong support for earthquake prediction, national defense, scientific research, and other fields.

To accurately measure subtle variations in magnetic field strength, direction, and other parameters, it is necessary to deploy multiple three-component sensors at different locations. Due to limitations in manufacturing processes, measurement errors may be introduced into geomagnetic measurements, which can affect the accuracy of the results. To address this, noise in the data can be processed beforehand. First, the characteristics of the noise, such as white noise, colored noise, etc., and its distribution across different frequencies, should be analyzed. Based on these characteristics, methods such as filtering algorithms (e.g. Kalman filtering, wavelet filtering) or machine learning techniques can be used to remove as much of the noise as possible, thereby improving the accuracy and reliability of the measurements.

Additionally, when measuring the magnetic field of an object *W*, it is necessary to place a geomagnetic monitoring sensor at a position far from *W*, while other sensors should be placed close to *W* for measurement. Since the values measured by the sensors include both the magnetic field of *W* and the geomagnetic field, the sensor measurements should be corrected by subtracting the values obtained from the monitoring sensor. This allows the true magnetic field of the object to be obtained. Establish mathematical models to solve the following problems.

**Problem 1:** Based on the data in Appendix 1, establish a suitable mathematical model to ensure that the variation in the measurement data from all sensors between 24:00 and 4:00 (midnight to 4:00 AM) is consistent. The initial values may differ due to the different positions of the magnetic sensors, meaning that the ,, and components of all magnetic sensors should each be adjusted by subtracting their respective initial values. Select a variation  from one sensor, such that the difference between the variation of each sensor and  (a process referred to as interference suppression) is minimized, ideally approaching zero.

**Problem 2:** Based on Problem 1, establish a suitable mathematical model to select a variation from one sensor, such that the variation in the measurement data of all sensors over a 24-hour period is consistent.

**Problem 3:** Based on the data in Appendix 2, each sensor measures the geomagnetic daily variation, with an additional signal from a specific object starting at a certain time (this signal changes over time). Sensor 1 is placed at a distance from the object and can be used as a monitoring sensor for geomagnetic fluctuations. Establish a mathematical model to identify the variation of the signal from the specific object.

**Note 1**. Your PDF solution of no more than 25 total pages should include:

• One-page Summary Sheet.

• Table of Contents.

• Your complete solution.

• References list.

• AI Use Report (If used does not count toward the 25-page limit.)

There is no specific required minimum page length for a complete MCM submission. You may use up to 25 total pages for all your solution work and any additional information you want to include (for example: drawings, diagrams, calculations, tables). Partial solutions are accepted. We permit the careful use of AI such as ChatGPT, although it is not necessary to create a solution to this problem. If you choose to utilize a generative AI, you must follow the COMAP AI use policy. This will result in an additional AI use report that you must add to the end of your PDF solution file and does not count toward the 25 total page limit for your solution.

**Note 2**. Similar to the approach in Problems 1 and 2, first calculate the difference between the variation of all sensors and the variation of Sensor 1. When the signal from the specified object is not present, the result of the difference should be as close to zero as possible. When the signal from the object is present, the result of the difference should correspond to the signal from the object.

**Note 3**. The first two columns of Appendix 1 represent the date and time, while the last three columns contain the , , and coordinate data. The first column of Appendix 2 represents the simulated time variable, and the last three columns contain the , , and coordinate data.